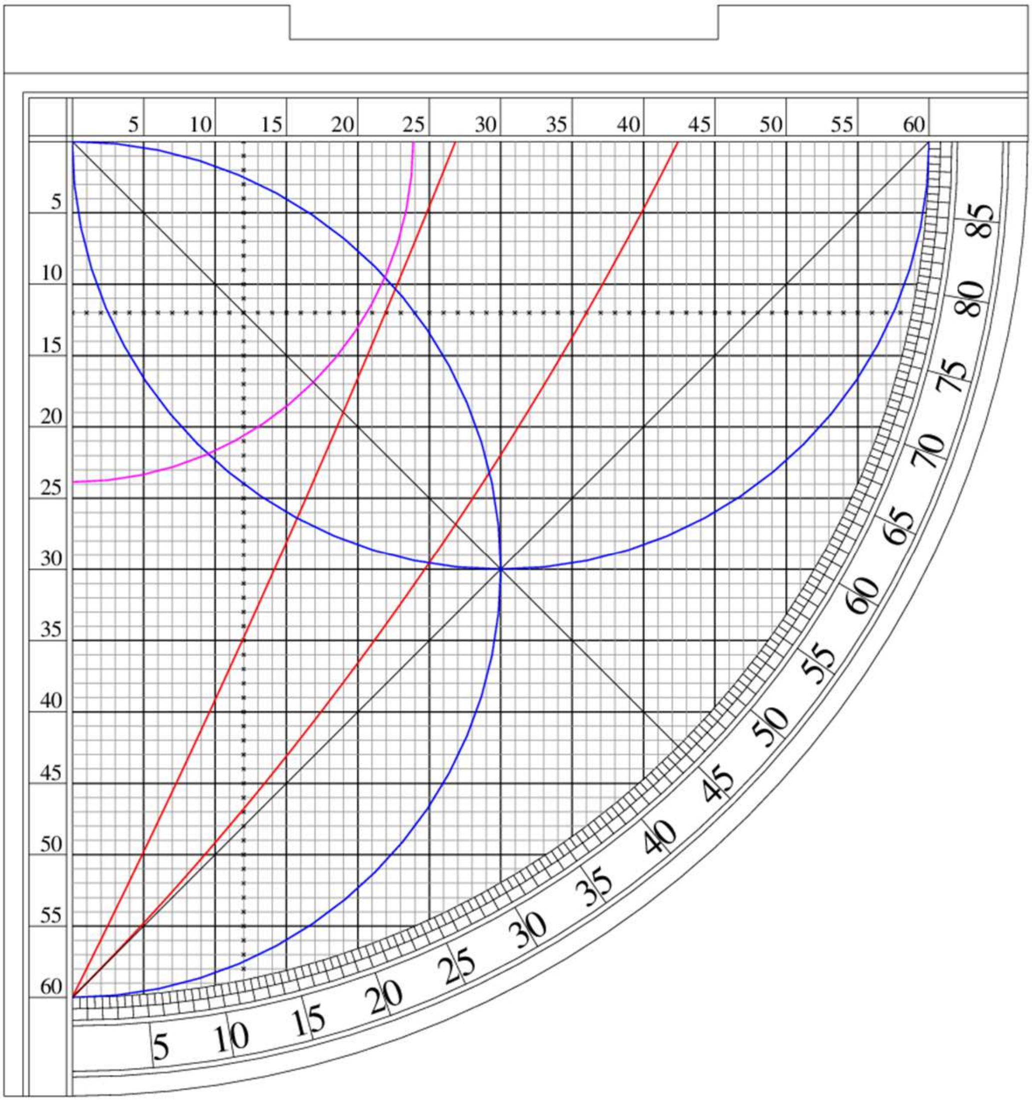
The Sine Quadrant in Theory and Practice



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Acknowledgements

Figure 1: Public domain from Wikimedia Commons:
http://commons.wikimedia.org/wiki/File:Unit_circle_trigonometry.svg

Figure 3: Ship art public domain from Wikimedia Commons:
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This is a handout for my class "The Sine Quadrant in Theory and Practice" first given at Pennsic War 43

Copies of this handout are available on my website at www.astrolabeproject.com

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Introduction

The development of mathematics and the sciences through-out the medieval period spawned a range of clever, elegant tools for observation and computation. Devices such as the Armillary Sphere and the Astrolabe are familiar sights in the art of both Europe and the Middle East; but lesser known are a class of tools known as quadrants.

Quadrants come in several different varieties, each with their own uses:

Astrolabe Quadrants "fold up" the lines and scales of an astrolabe into a compact device that can perform most of the same calculations.

Horary Quadrants deal with telling time and converting between differing time keeping systems.

Finally, the Sine Quadrant, the subject of this class, allows the user to perform quick, accurate trigonometric calculations; along with several other related functions.

The sine quadrant (also known as the Sinecal, or Rubul Mujayyab), first appeared in the Middle East and Persia prior to the 10th century[1]. Its use spread, and while it is lesser known in Europe, several extant examples of European versions of the device do survive. The device is still in use in parts of the world today.

Materials:
The materials used to construct these devices vary widely. There are surviving examples in various metals, wood, bone, ivory, and paper. Fancy ones exist, as do simply made ones.

Math Review
(Sorry about that)

Before we go on, I will quickly review some basic trigonometry that will be needed to follow what we will be covering.

In mathematics, the trigonometric functions are functions of an angle. They relate the angles of a triangle to the lengths of its sides.

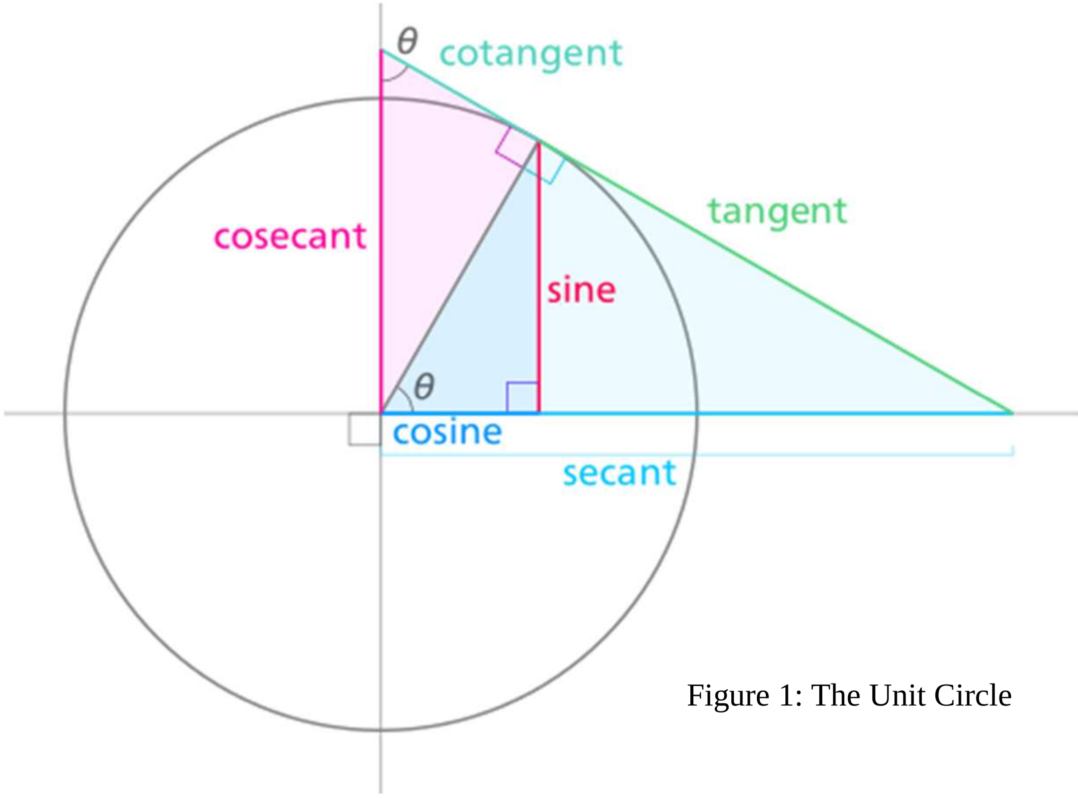

Figure 1: The Unit Circle

Trigonometric functions are important in surveying, geometry and many other tasks. The most familiar trigonometric functions are the sine and cosine functions.

For a given angle a in a right triangle, the Sine is the ratio between the length of the opposite side to the hypotenuse side (SIN = O/H). Cosine is defined as the ratio of the adjacent side to the hypotenuse side (COS = A/H).

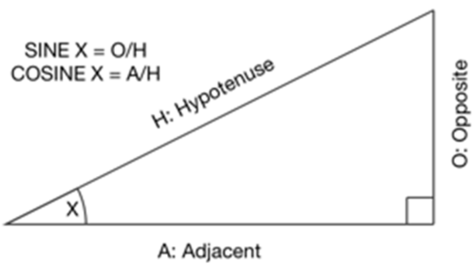

Figure 2: Sine and Cosine

Math Review

(Sorry about that)

Sine and cosine are the basic functions in trigonometry, all the other trig fictions can be expressed as combinations of those two:

> Tangent = o / a → Tangent a = Sine a / Cosine a
> Cotangent = a / o → 1 / Tangent → Cotangent a = 1 / Tangent a
> Secant = h / a → 1 / Cosine → Secant a = 1 / Cosine a
> Cosecant = h / o → 1 / Sine → Cosecant a = 1 / Sine a

The point here is that if you are given two of the three numbers in one of these relationships, you can compute the third.

Example:
Your ship is resting at anchor with 137 feet of taught anchor rope out at an angle of 32 degrees.

How deep is the water?

We know the hypotenuse and the angle and want to find the length of the opposite side so sine is what we want.

Sin = Opposite/Hypotenuse

So

Sin 32 = x/137

Becomes

x = 137 * Sin 32

Sin 32 is .53,

So x = 137 * .53

Or 72.6 feet

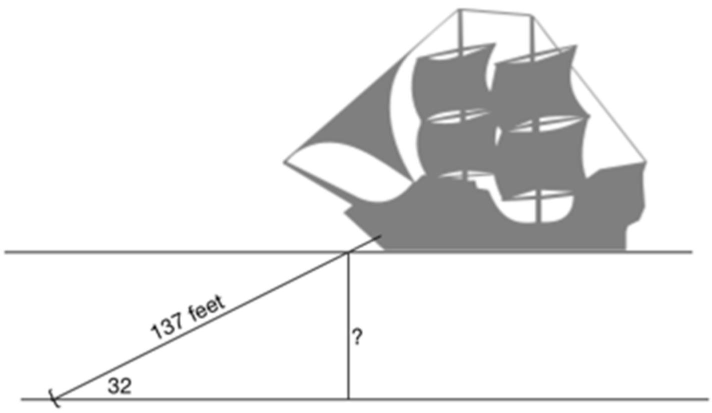

Figure 3: An Example

Basic Features of the Sine Quadrant

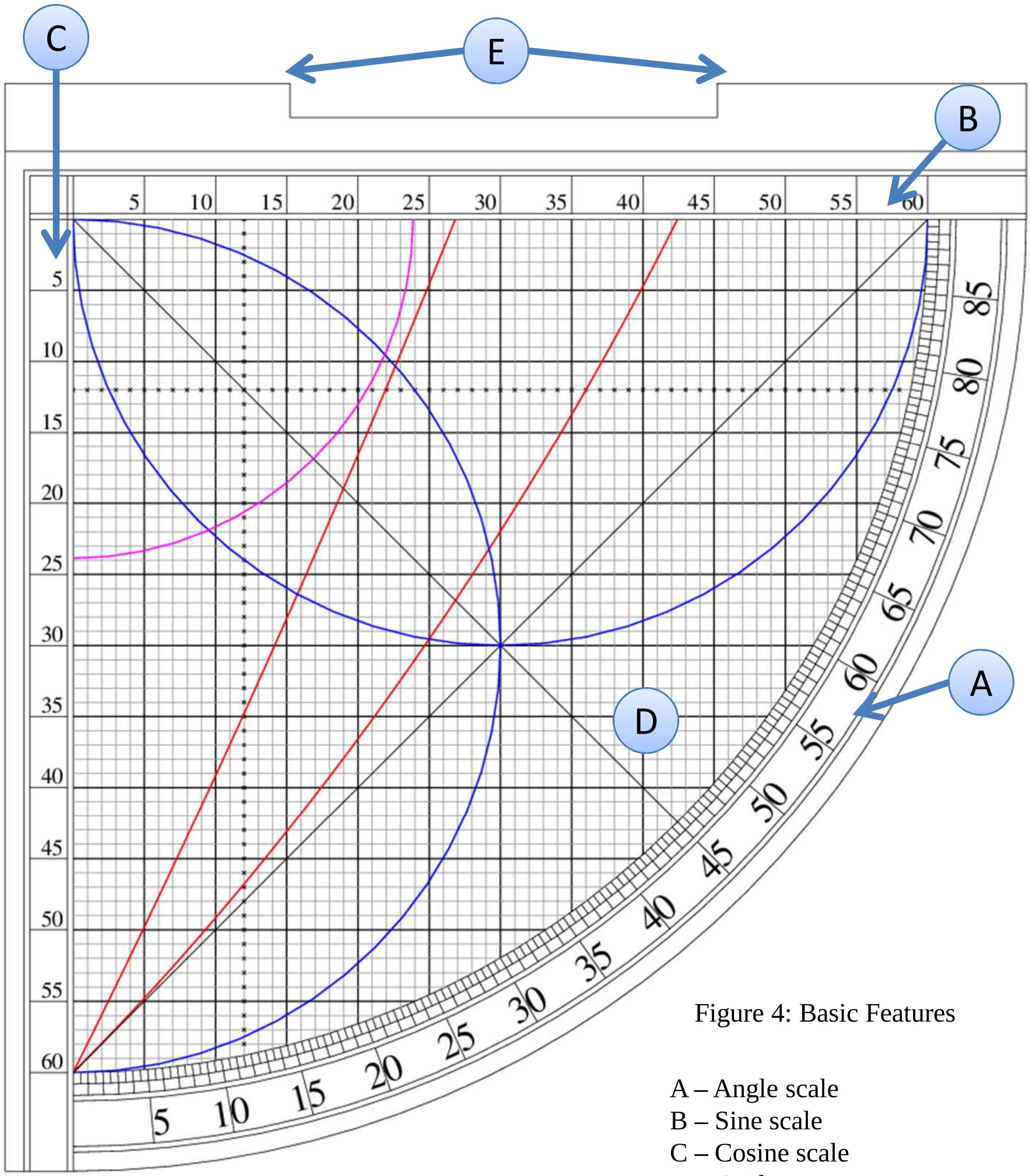

Figure 4: Basic Features

A – Angle scale
B – Sine scale
C – Cosine scale
D – Grid
E – Sights

The Basic Features

The Basic Features:
Most quadrants are shaped, logically enough, as a quarter-circle, with two straight sides meeting at a right angle and a quarter-circle arc closing the open end .

The Angle Scale
Along the curved side of the quadrant is a scale marked off in 90 degrees, usually grouped into five degree sections. Used with the sights and the plumb cord, the user can sight on a target and determine the angle of elevation by seeing where the weighted cord lies on the angle scale. In your example the scale has been simplified, but in actual quadrants from period the angle scale was often marked in both directions, That is 0-90 degrees and 90-0 degrees. This would allow the user to measure angles from both the horizon and zenith easily (A).

The Horizontal and Vertical Scales
Along each straight edge of the quadrant is a scale. These are traditionally divided into 60 units and subdivided into 12 five unit sections. This base-60 (sexagesimal) numbering is a hold-over from the Babylonian number system and can be seen also in our divisions of time and angles. The horizontal scale is used for computing the Sine of an angle, the vertical scale is used to compute the Cosine of an angle (B, C).

The Sine/Cosine grid
On the face of the sine quadrant there is a 60 by 60 grid matching the horizontal and vertical scales discussed above. As on the scales, every fifth line is often set apart, either by width or with special markings or inlay (D).

The Sights
On almost every quadrant there is a set of sights. Sometimes these are sighting vanes with holes for sighting through; sometimes a square notch is cut out of one side, providing two posts that can be sighted across (E).

The Cord
At the right-angle of the quadrant there is a small hole from which hangs a weighted cord. This cord is used as an index line when doing calculations. It also works as a plumb line when the quadrant is used to measure altitudes. Often there is a bead placed on the cord, fitted so that it can be moved back and forth to mark locations on the cord (not shown).

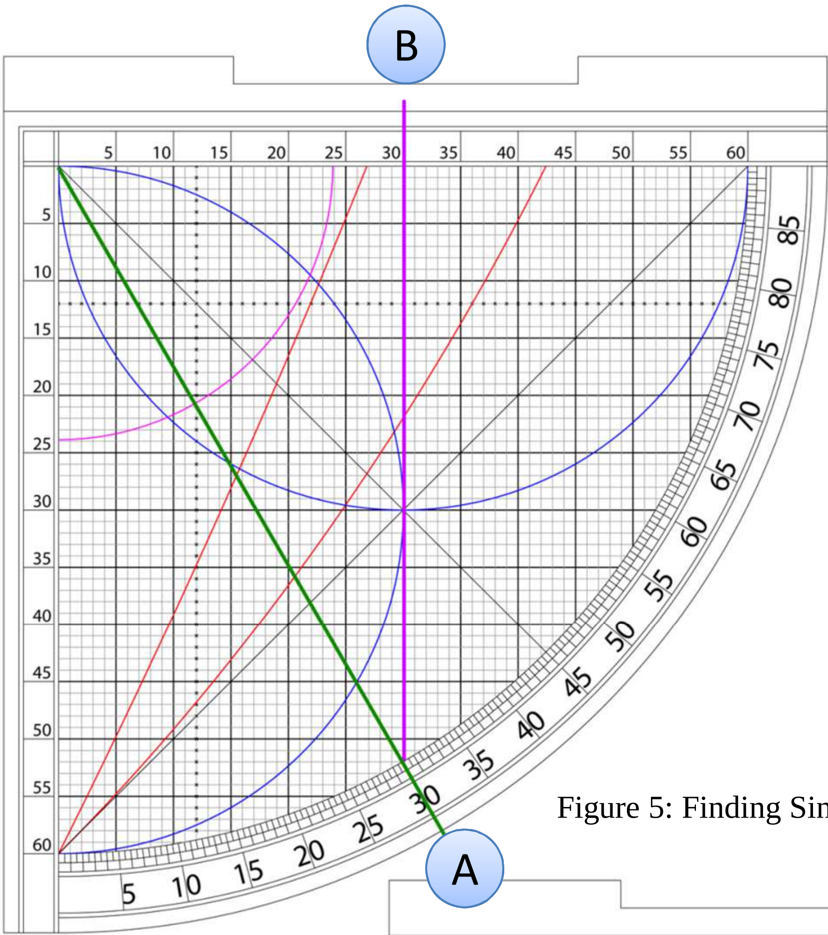

Figure 5: Finding Sine

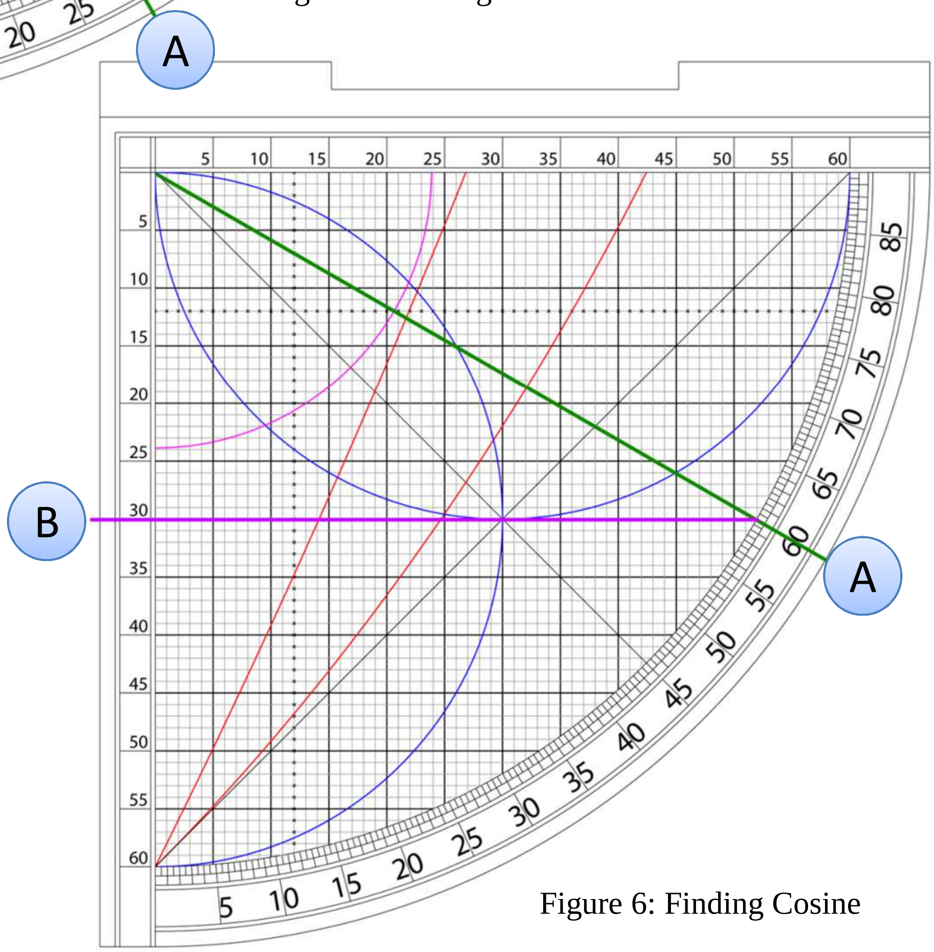

Figure 6: Finding Cosine

Basic Functions

Measuring angles
The most basic function of the Sine Quadrant, common to all quadrants, is measuring angles. By using the sights, the weighted cord, and the angle scale together, a user can determine a vertical angle with a good amount of accuracy.

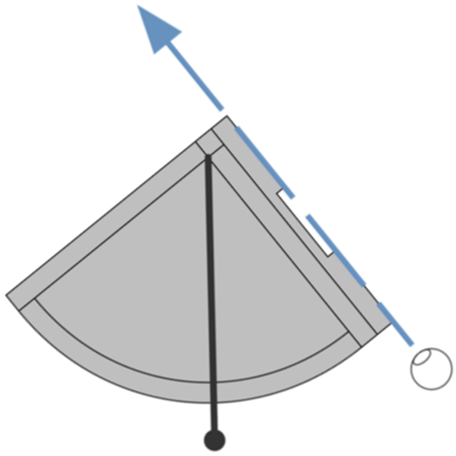

Figure 7: Sighting

By sighting carefully on the target through the vanes, or across the top of the notch, you can get an accurate angle to the target by reading the position of the cord on the angel scale.

Note: If you are taking a sighting on the Sun, as is often the case, do not look directly at it. instead, use the shadow of the front sight against the back sight to determine alignment.

Finding the Sine and Cosine of an Angle
It is often necessary to find the Sine or Cosine of an angle when performing a calculation. Finding the rough figure for Sine or Cosine for a given angle is easy using a Sine Quadrant.

For example: Given the angle of 30 degrees, find the sine [Figure 5].

- Holding the quadrant, move the cord until it is held taught on the 30 degree mark (A).
- Next look at the point that the cord crosses the curved edge of the grid.
- Follow that point vertically until you reach the horizontal sine scale, read the result (B).

An angle of 30 degrees gives us the correct sine of 30/60 or 0.5.

Computing Cosine is done in a similar manner [Figure 6], but using the vertical scale in place of the horizontal sine scale. Notice that as the cord is rotated from 0 to 90 degrees the sine varies from 0/60 (0) to 60/60 (1) with the Cosine changing in reverse from 1 to 0, as expected.

To find the angle represented by a sine, the process works in reverse: Given a sine of .5 or 30/60, trace the 30 line down to the rim of the grid, place the cord there and read the angle.

That covers the basic functionality of the sine quadrant. But there is much more to explore.

Advanced Features

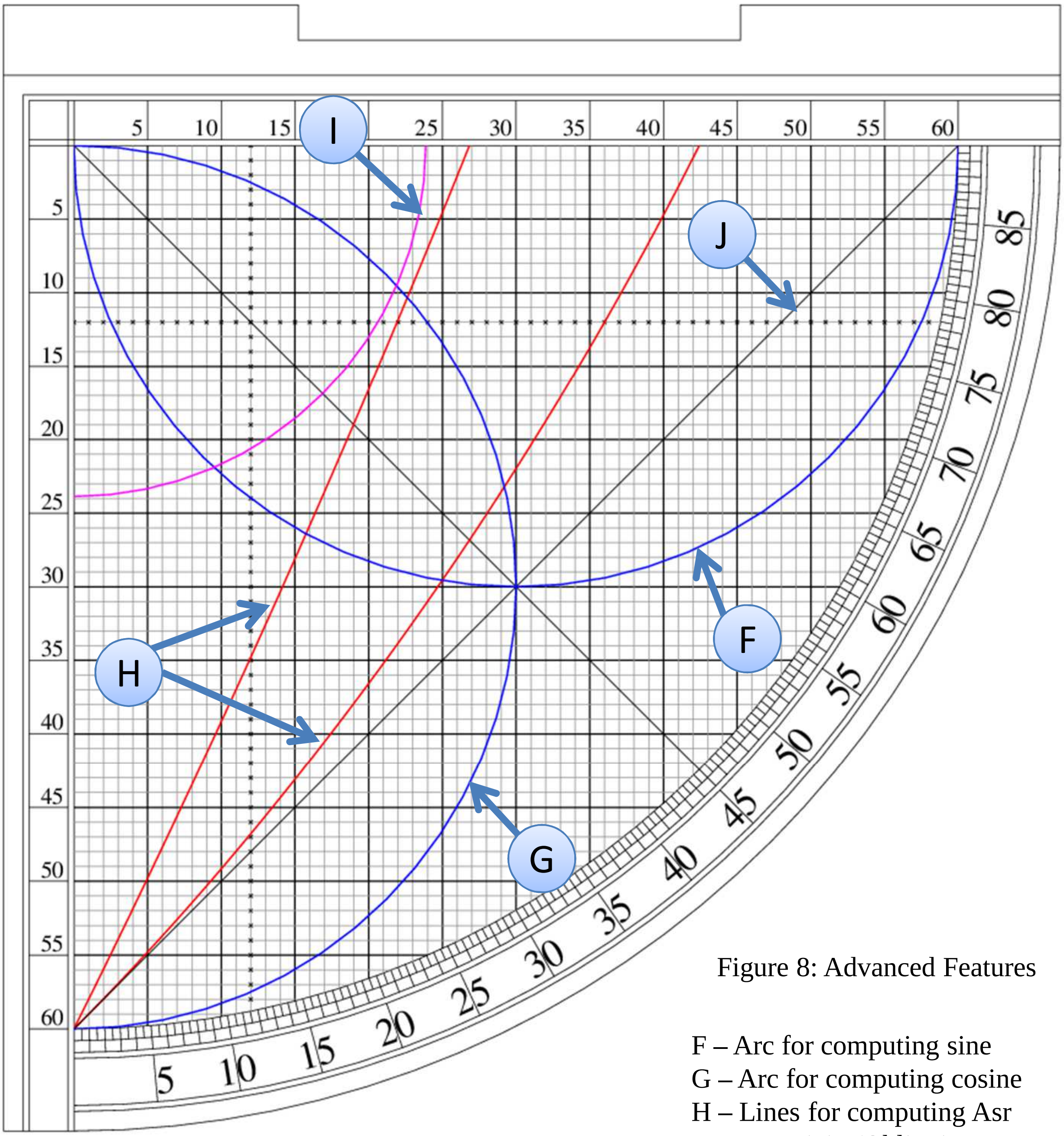

Figure 8: Advanced Features

F – Arc for computing sine
G – Arc for computing cosine
H – Lines for computing Asr
I– Eccentricity/Obliquity arc
J – (Unknown Function)

Advanced Features

On most, if not all examples I have seen, there are several lines marked into the grid that are used for advanced functions [Figure 8].

The Sine and Cosine Arcs

On many sine quadrants there will be two half-circle arcs (blue on your example), one centered on the sine scale (F), one centered on the cosine scale (G). These can be used in conjunction with the sine and cosine scales as an alternative method of converting angles to sine/cosine.

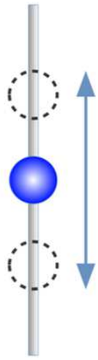

Figure 9: Bead

Note: If these arcs are to be used, there has to be a moveable bead on the weighted cord, this bead is used as a cursor to mark a position on the cord. Think of it as memory storage for the device.

To use these arcs, the procedure is similar to using the grid. First the user pulls the cord taught over the desired angle. Next, the user slides the index bead to rest directly on the appropriate arc (the horizontal arc for sine, the vertical for cosine). Once the marker is in place, the user will then rotate the cord to the sine or cosine scale (either will work for this) and read the answer from the point under the bead.

Advanced Functions

For example, see the figure below:

The cord is first set to 30 degrees (A), the bead is then positioned directly on the sine arc(B), then the cord is rotated to the horizontal sine scale (C) and the sine is read (D) as 30/60 or 0.5.

Converting to other way, from sin/cosine back to an angle is straightforward as well. The user just lines the cord up with the scale, positions the bead at the given sine or cosine; then rotates the cord until the bead touches the appropriate arc. The cord will be set to the equivalent angle.

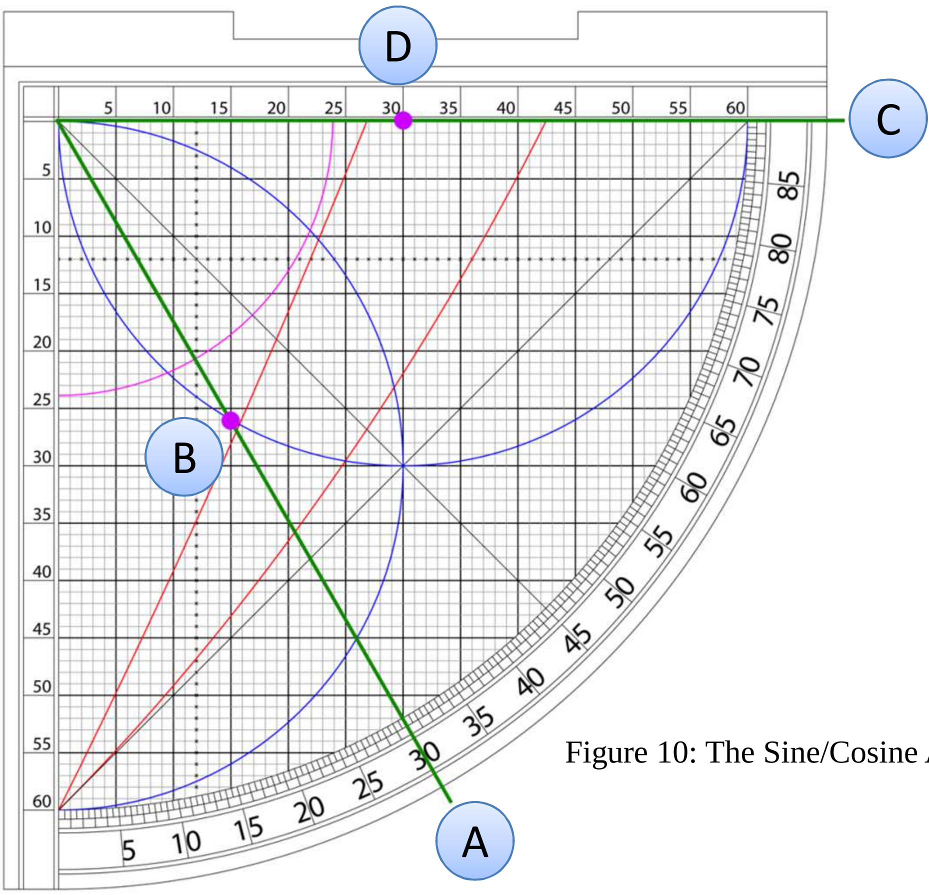

Figure 10: The Sine/Cosine Arcs

Advanced Functions

The Obliquity Arc

Another line found on many examples of the Sine Quadrant is a circular arc centered on the origin point at the quadrant's right angle [Figure 8, I]. With a radius of approximately 24 units (marked in purple on your example) this arc is a projection of the Earth's orbital obliquity (the tilt of the Earth's rotation to its orbital plane). The purpose of this marking is to allow the user to determine the Sun's declination (angle above or below the equator) for any given day, allowing the user to then determine the Sun's altitude at noon for that day (besides being neat, this will be of use later on.)

To understand how this works we will need to review a bit of basic astronomy:

The Earth's axis of spin, and therefore its equator, is tipped 23.4 degrees to the plane of the planet's orbit.

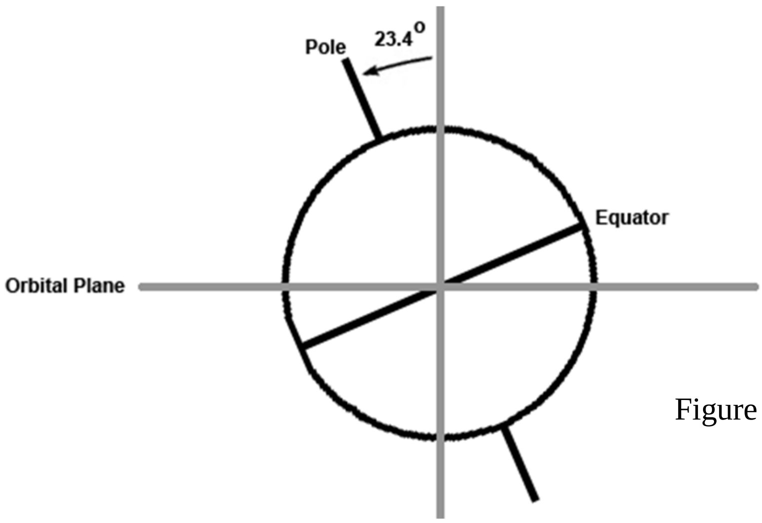

Figure 11: Orbital Tilt

This means that as the Earth moves around the Sun in the course of a year, the Sun appears to move back and forth over the equator spending time in both the Northern and Southern hemispheres.

Advanced Functions

Traditionally, the Sun's path through the sky (the ecliptic) is divided up into 12 30-degree zodiac signs. The Spring Equinox defines the zodiac's starting point, the "First Point of Aries" (Aries 0), when the Sun is directly over the equator getting ready to head north. At this point the day and night are of equal length. Each day the Sun moves a little way along the ecliptic, progressing around the zodiac until it returns to its starting place. As the days pass the Sun appears to creep north until the Summer Solstice, when the Sun is at its northern-most point and the day is at its longest. Then the Sun moves back south. The movement of the Sun and the seasons should be familiar to all.

Now look at the following diagram:

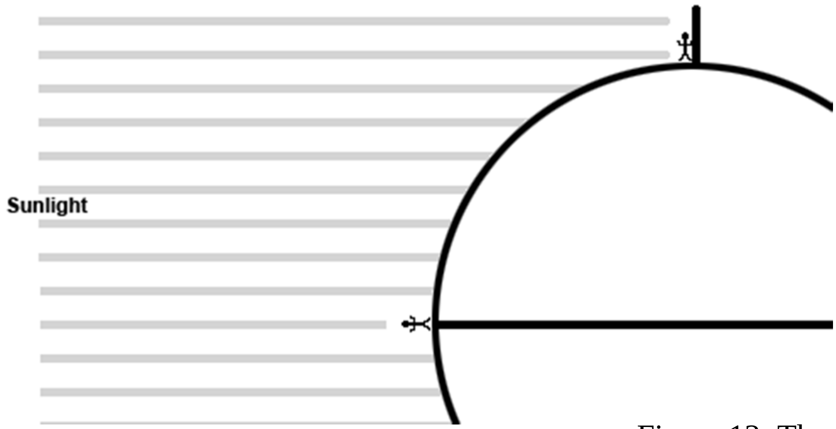

Figure 12: The Sun at Equinox

At the equinox the sun shines directly above the equator, so for someone standing on the equator, at noon the sun would be directly overhead, that is, at an angle of 90 degrees to the horizon. If another person was standing at the North Pole at the same time, they would see the Sun on the horizon, or at 0 degrees elevation. Therefore, you can compute the angle of the Sun above the horizon for the equinox as:

$$\text{noonAngle} = 90 - \text{latitude}$$

When the Sun is at a different part of the ecliptic, it will be up to several degrees north or south of the equator, so the equation becomes:

$$\text{noonAngle} = (90 - \text{lat}) + \text{declination}$$

Or to put it another way, you can compute the Sun's noon altitude if you subtract your latitude from 90 and then add the Sun's declination for that day.

Advanced Functions

Example: Given you are at 50 degrees north latitude, find the Sun's noon altitude for the Summer and Winter solstices (remember that the Sun is 23.4 degrees from the equator at solstice):

Summer: (90-50) + 23.4 = 63.4 degrees
Winter: (90-50) + -23.4 = 16.6 degrees
(remember the Sun has a negative declination in winter/south of the equator)

Returning to our sine quadrant now: How can we determine the Sun`s declination for a given day?

The zodiac is divided up into 360 degrees. At the Spring Equinox the Sun is at 0 degrees on the zodiac; at the Summer Solstice the Sun is at 90 degrees; at the Fall Equinox, 180 degrees; Winter Solstice sees the Sun reach 270 degrees and it finally returns to 0 at Spring Equinox again.

Now take the sine quadrant and hold the cord at 0 degrees. Note where it crosses the obliquity arc and follow the grid down and read the angle: 0 degrees (A). Now move the cord to 90 degrees (B) and do the same: you will get an angle of about 23.4 degrees (C).

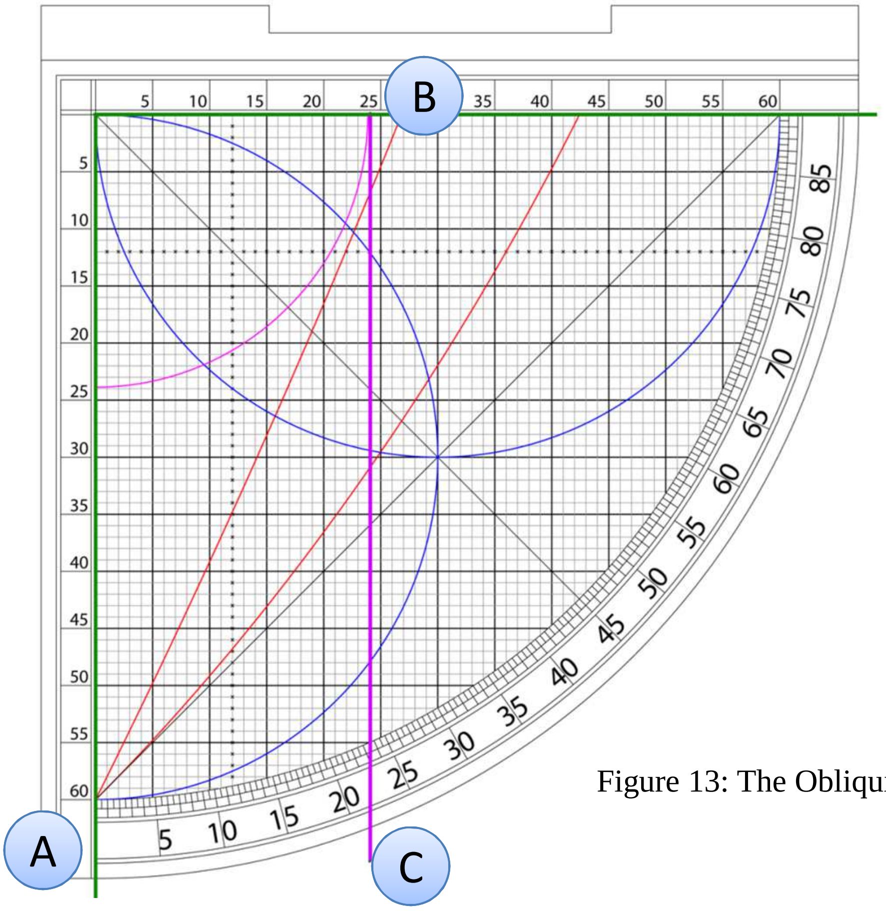

Figure 13: The Obliquity Arc

Advanced Functions

At this point reverse the direction you move the cord and move an additional 90 degrees to 180 (the cord is now back at 0). By moving the cord up and back four times you sweep out 360 degrees, and can simulate traversing the entire zodiac. Zero degrees represents the Spring and Fall equinoxes, and 90 degrees represents the Summer and Winter solstices.

So, let us say you want to know the angle of the Sun at noon for the 5 of May. If you look up the Sun's position for that date in your ephemeris (if you needed to use this device you would most likely have one hanging around somewhere), you get a figure of Taurus 15. Taurus is the 2nd symbol in the zodiac, so add 15 degrees to the 30 for the first symbol (Aries); this gives us 45 degrees. Move the cord to the proper position, 45 degrees (A), and mark where it crosses the obliquity arc (B), follow this point down to the degree scale and read off the Sun's declination as 16.5 (C). You are still standing at 50 North (from the previous example), so you can compute the Sun's elevation at noon for that day to be:

$$(90-50) + 16.5 = 56.5 \text{ degrees}$$

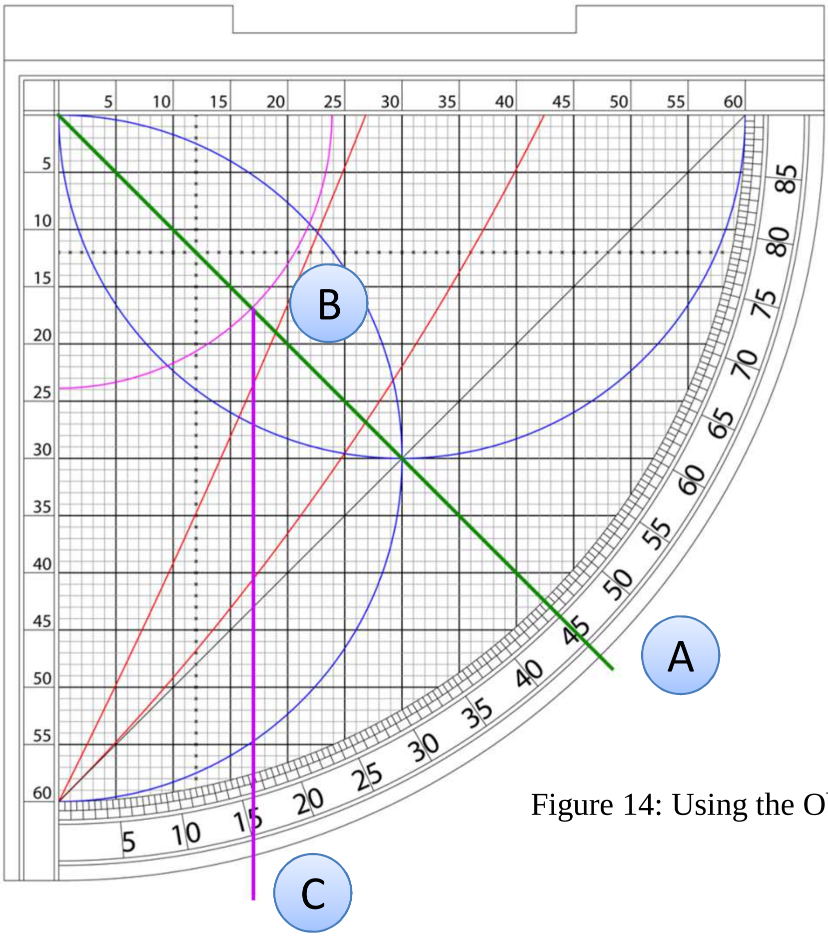

Figure 14: Using the Obliquity Arc

Advanced Functions

So, what if you misplace your ephemeris? Then what?

Well, the calendar and the zodiac don't match up well, there are 365.25 days in a year, and 360 degrees in a circle. In addition, the sun moves through the Zodiac faster or slower depending on the time of year.

But, counting one day per degree puts us off only 5.25 degrees by the end of the year, this would translate to an error in declination of less than two degrees. If we are counting days since the last solstice or equinox, the possible error is only a quarter of that, probably within the observational error of the instrument. So, depending on how important fine accuracy is to you, you might not need an ephemeris at all.

Let's rework the last example without an ephemeris:

The Spring Equinox is March 20th, therefore the 5 of May is 46 days later. So moving 46 degrees around from 0 (Spring Equinox) we place the cord at 46 and read a declination of 17 for the Sun. This is only slightly off (half a degree) from the figure we got above (16.5).

So. as we have seen, with a little mental calculation, a person can use a sine quadrant to find the angle of the Sun at noon for any day of the year.

The obvious next question is why would you need to know?

Advanced Functions

The Asr Lines
One of the major uses of a sine quadrant was computing the proper times for the Muslim mid-afternoon prayer, Asr[2].

Prayer Times
Traditionally the times set for the 5 daily prayers required by the Muslim religion are based on the sun's position in the sky[3]. Of interest to us today is the mid-afternoon prayer, Asr. The period for Asr begins when the shadow of a vertical pole is equal to its noontime length plus the length of the pole; and ends when the shadow is the noontime length plus twice the length of the pole.[4]

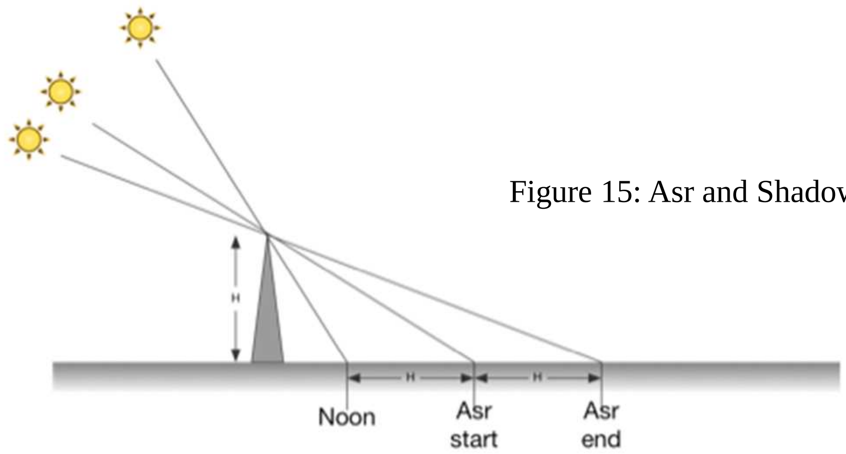

Figure 15: Asr and Shadows

Because Asr is based on the Sun's shadow, the time will vary depending on location and date; so determining the proper times for prayer must be done for each day. The sine quadrant allows the user to determine these times accurately and quickly by use of specialized lines on the face.

If you look at the example quadrant, [or Figure 9 H], you will see a pair of shallow (almost straight) diagonal curves, marked in red. The curves run from the zero degree mark up to roughly the middle of the horizontal scale. The lower line is used to compute the start of Asr, the upper (often missing) allows you to compute the end.

By the definition above, both times are based on the Sun's position at Noon; This gives us one use for finding the Sun's noontime altitude. Once the angle of the Sun at local Noon is found, it is just a matter of marking where that line crosses the two Asr curves, to determine the Sun's angle at the beginning and end of Asr.

Advanced Functions

For example: Let's work through finding the times for Asr for 31 degrees North (just south of the city of Alexandria, Egypt), for the 15th of February.

First, determine the Sun's angle at noon:
The 15th of February is 56 days after the Winter Solstice on December 21. So, remembering that the solstices are at the 90 degree mark of the angle scale, we count down 56 degrees and place the cord at 34 degrees (A). Make a note of where the cord crosses the obliquity arc (B), and follow that point down to get an angle of 13 (C).

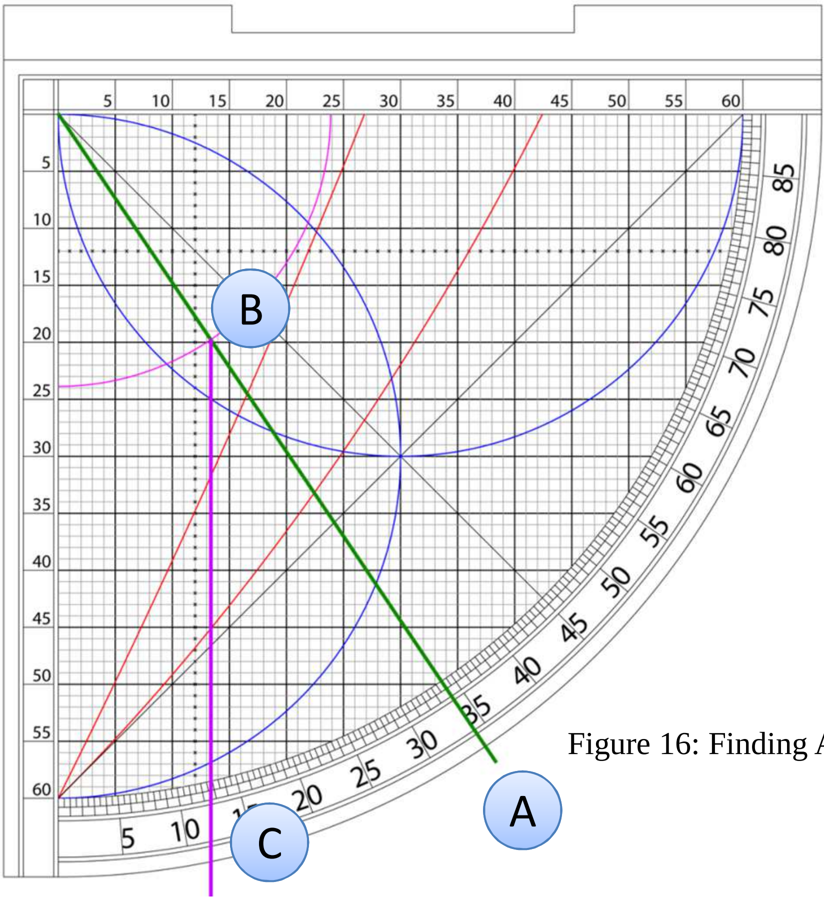

Figure 16: Finding Asr First Step

Remember that this is Winter, so the Sun is in the South and therefore the declination is negative, giving us a declination of -13. Using that and our known latitude we compute the Sun's angle at noon to be:

$$(90-31) + -13 = 46$$

Advanced Functions

Now that we know what the Sun's angle at Noon will be, we can determine its angle for the beginning and end of Asr very easily using the Asr lines:

Place the cord on the computed Noon angle (A), and note where the cord crosses the first Asr line (B); follow this point down to the angle scale to find the angle of the sun at the start of Asr (C). Now note where the cord crosses the second Asr line (D), and do the same to find the angle of the Sun at the end of Asr (E).

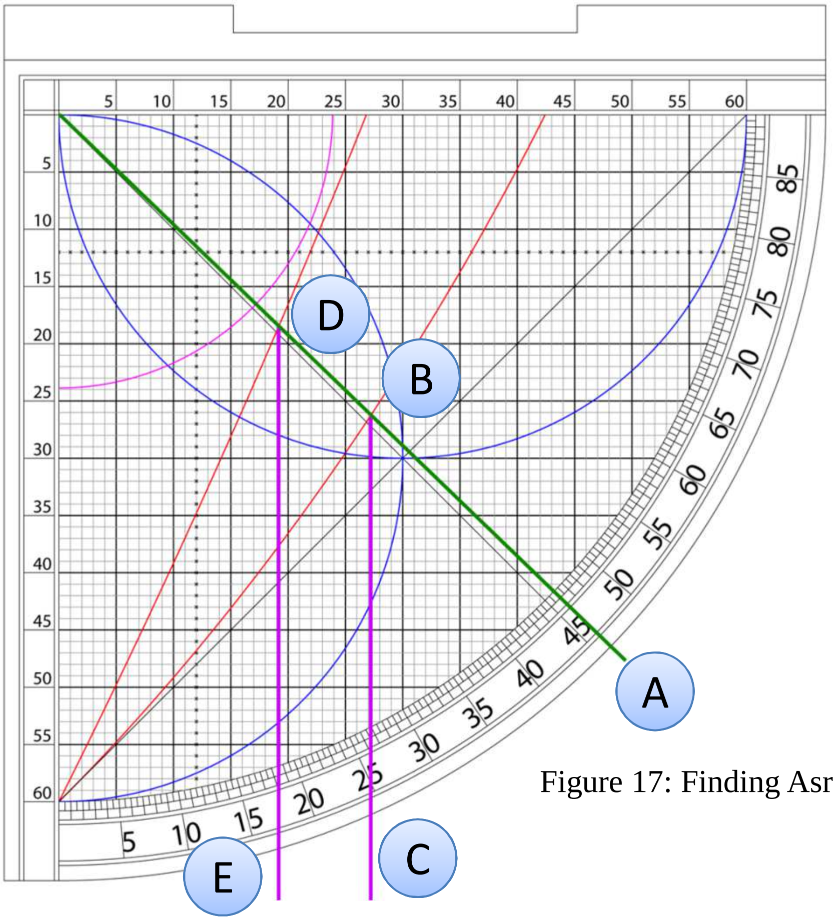

Figure 17: Finding Asr Second Step

So we can see that on the 15th of February in the vicinity of Alexandria Asr starts when the sun is at 27 degrees above the horizon, and ends when the sun sinks to 18.5 degrees above the horizon.

The Other Asr Lines
The two arcs described previously are one method of determining the times for the midafternoon Asr prayer, but there are alternative methods.

Some sine quadrants do not have the arcs, but rely on a different set of markings entirely. Depending on the maker, the device may have one set of markings or the other, or both, or none.

If you look at the example sine quadrant you will see, in addition to the previously discussed Asr arcs, a line of markings (marked with black dots) parallel to the horizontal and vertical scales at 12 units [see Figure 18].

These lines are most often marked at 12 units, but I have seen several examples with an additional set of markings at 7 units. It does not matter which set are used, as the answer they give is the same.

The vertical line at 12 units can be used to simulate a pole 12 units high[5]. By marking where the cord crosses the line when it is set to the Sun's noon angle you know the length of the shadow at noon. Then by adding 12 units and moving the cord to cross at that point, you can compute the angle of the Sun at the start of Asr. Finally, moving the cord down a further 12 units will give you the angle for the end of Asr [Figure 19].

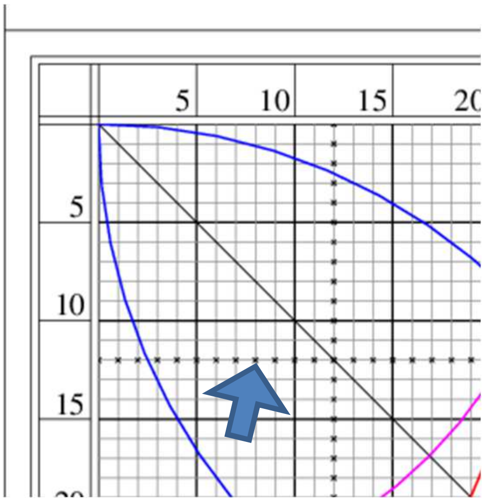

Figure 18: Other Asr marks

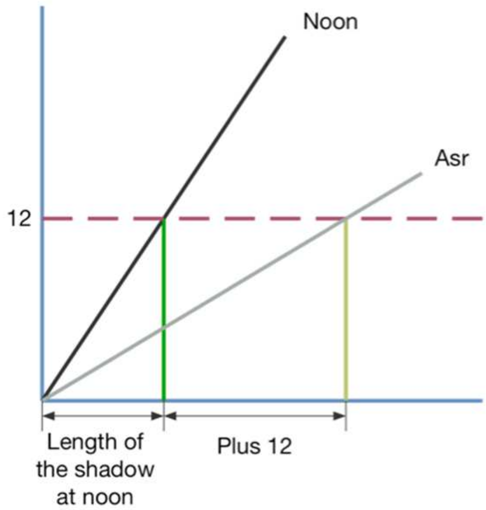

Figure 19: Using the Asr lines

Advanced Functions

To demonstrate that using the Asr arc and the Asr line both give the same answer, let's work through an example where the Sun is at a height of 50 degrees at local noon.

First set the cord of the quadrant on the 50 degree mark (A). Note where it crosses the Asr arc at (B), and follow that line down to find an angle of 28.5 degrees. Now notice where the cord crosses the 12 unit Asr line at 10 units (C). add 12 units to this (the "height" of the "pole"), to get 22 (D) and move the cord to cross at this point. The cord will lie at the expected angle, 28.5 degrees (E).

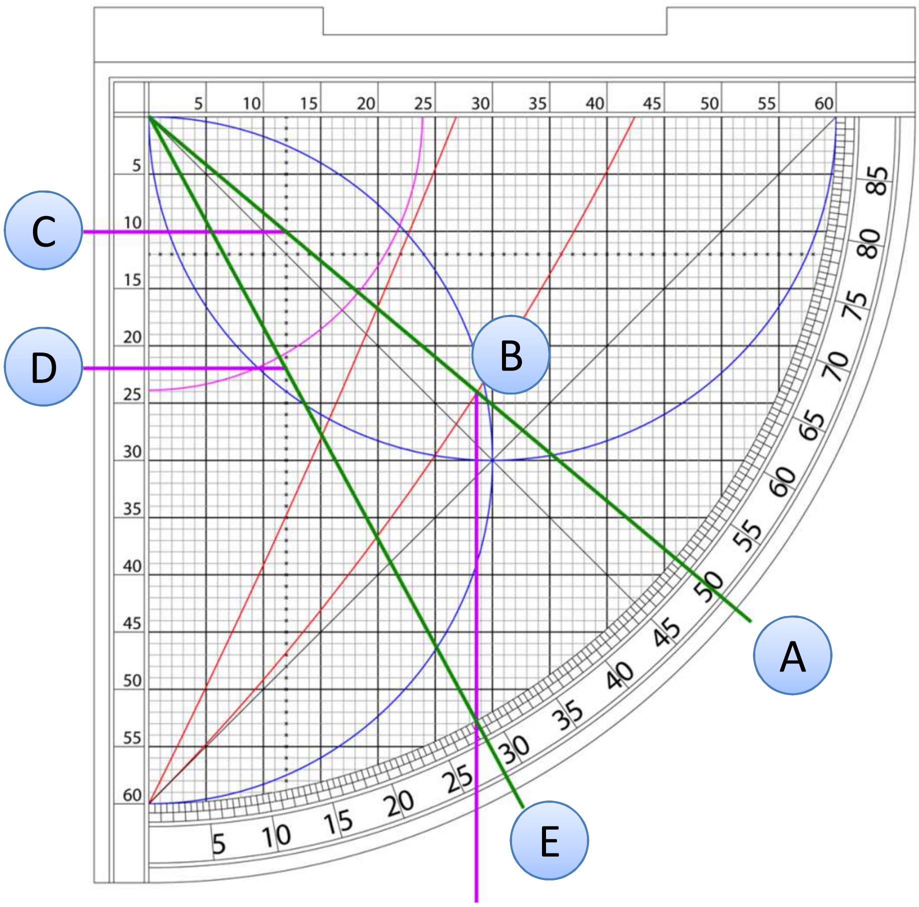

Figure 20: Using the Asr lines

To find the angle of the Sun at the end of Asr, you just need to add an additional 12 units to simulate adding twice the length of the pole.

Advanced Functions

Estimating Mecca

For a Muslim, knowing the direction to Mecca from one's current position is not just important because of the requirement to pray in that direction five times daily. In addition, the ritual butchering of food animals needs to be done with that direction in mind for the meat to be acceptable to the dietary laws; and when bodies are buried, they need to be placed on their sides, facing toward Mecca[6]. The location of the Kaaba, the focus of prayer, is 21.4N 39.8E[7]. Determining the direction, known as the Qibla, to that point is a major subject in medieval Islamic mathematics[8].

To complicate matters, the Qibla is defined as being the "Great Circle" direction between a given point and the location of Mecca[9]; and there are many treatises devoted to computing it accurately. But in the case of a traveler, without access to intricate mathematics or carefully computed tables, finding the proper direction is tricky.

It is, however, possible to estimate the local Qibla using a sine quadrant. To do this, the user must know the latitude and longitude of Mecca, as well as their own current latitude and longitude[10].

The problem is worked as follows:

- First, find the difference in latitude between the two positions
- Next, find the difference in longitude.
- Set the cord to point at an angle from horizontal equal to the difference in latitude. Mark the line from there to the sine scale mentally.
- Set the cord to point at an angle from vertical equal to the difference in longitude. Mark the line from there to the sine scale mentally.
- Finally, set the cord to lay across the point where the two lines cross. The line will now point to a direction. You will need to determine whether the direction given is to the North, South, East, or West. For example, if you are North and West of Mecca, the Qibla direction will be measured from South to the East [Figure 21].

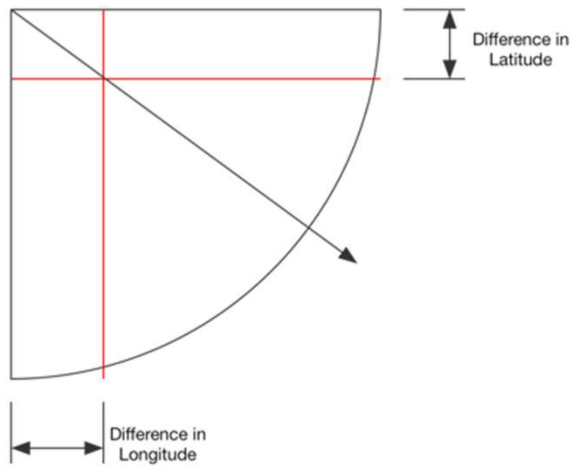

Figure 21: Estimating direction

Advanced Functions

Example:
Finding the Qibla from the vicinity south of Alexandria Egypt (location: 31.2N 30.0E)

The difference in latitude (rounding off) is: 10 degrees
The difference in longitude is: 10 degrees

Work the problem as below: Measure down ten degrees from the horizontal scale(A) for the difference in latitude(B), and ten degrees from the vertical scale(C) for the difference in longitude(D). Place the cord on the intersection of the two imaginary lines(E) and it will point at an angle(F), 45 degrees in this case. As Mecca is east and south of Alexandria, this angle will be the angle south of due East. So add it to 90 degrees to get the compass bearing (135 degrees). I tested this using the ruler function in Google Earth (which conveniently uses great circles) The great circle direction to Mecca is 135.68 degrees.

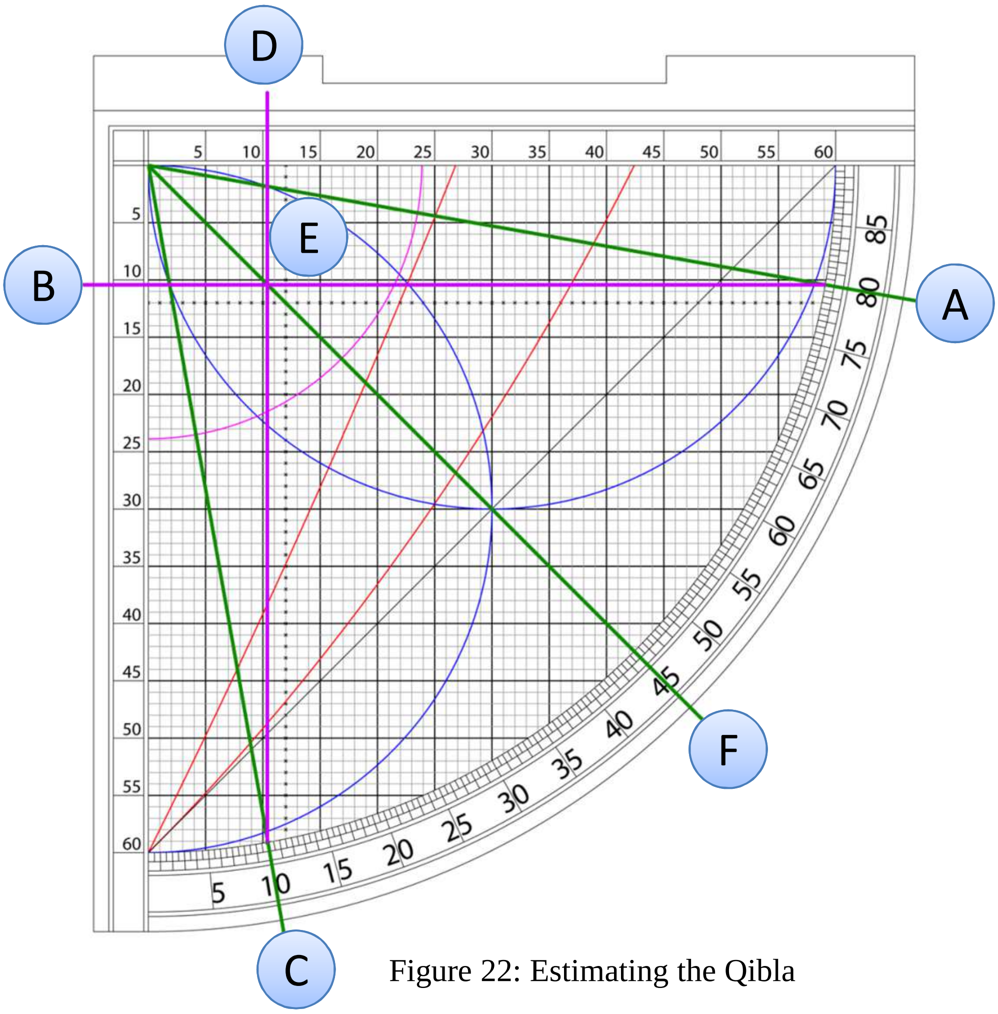

Figure 22: Estimating the Qibla

Advanced Functions

Limitations
Clever as this technique is, it is limited in usefulness. Because the Qibla is a great circle line between points on the globe, the further you are from Mecca, the more inaccurate the estimate will be. For example in the city of Casablanca (location:33.5N 7.5W) the Qibla direction is about 93.5 degrees but if you estimate it using the sine quadrant you get a direction of 106 degrees.

Additionally, this will not work for points further apart than 90 degrees latitude or longitude.

Note that this technique works just as well for finding the direction between any two points.

Summary
Some questions I still have:

1. Why do some examples of the sine quadrant have the Asr arcs reversed? On some examples I have seen , the arcs extend from the 90 degree point down toward the cosign (vertical axis) not from 0 toward the horizontal sine axis.
2. Why do many examples of the sine quadrant contain both a 12 unit and 7 unit Asr line? These give the same answer, there is no reason I can see for marking both. The traditional height of a gnomon in the Middle East is 7 feet, and the shadow square of an astrolabe often has a 7 unit side for that reason. But why 12 units then? Perhaps just because 12 divides up easily into halves, thirds and quarters?
3. Why are the 7 and 12 Asr lines always drawn on both the horizontal and vertical when only the vertical is used?
4. What is the purpose of the line that is often drawn as a chord from 0 to 90? on several examples I have seen the line is labeled (illegibly, alas), indicating some use, but what? I can find no reference.
5. I can see why the angle scale is often labeled in both directions (to allow measuring distance from zenith); but why are the sine/cosine scales also often marked in both directions?

Notes

Notes:
[1] King, David A. "Astrolabes, quadrants, and calculating devices ." Encyclopaedia of Islam, THREE. Edited by: Gudrun Krämer, Denis Matringe, John Nawas, Everett Rowson. Brill Online, 2013. <http://referenceworks.brillonline.com/entries/encyclopaedia-of-islam-3/astrolabes-quadrants-and-calculating-devices-COM_0033>

[2] Bir, Atilla. (2008). Principle and Use of Ottoman Sundials. Retrieved from http://www.muslimheritage.com/topics/default.cfm?ArticleID=942

[3] King, David A. A Survey of Medieval Islamic Shadow Schemes for Simple Time-Reckoning. Oriens, 32(1990), 196-197. http://www.jstor.org/stable/1580631

[4] Note: There are various schools of thought, and regional and cultural variations. The above is not definitive and is based on several sources.

[5] Charette, Francois, Mathematical Instrumentation in Fourteenth-Century Egypt and Syria. The Illustrated Treatise of Najm al-Din al-Misri, Brill, Leiden (2003). Pg 176-177

[6] King, David A. and Lorch, Richard P., “Qibla Charts, Qibla Maps, and Related Instruments”. The History of Cartography, Volume 2, Book 2: Cartography in the Traditional East and Southeast Asian Societies, University Of Chicago Press, 1995. pg 189.

[7] “Kaaba” http://en.wikipedia.org/wiki/Kaaba

[8] King, David A., World Maps for Finding the Direction and Distance to Mecca: Innovation and Tradition in Islamic Science, Brill:1999, pg 56.

[9] “Qibla” http://en.wikipedia.org/wiki/Qibla

[10] King, David A., World Maps for Finding the Direction and Distance to Mecca: Innovation and Tradition in Islamic Science, Brill:1999, pp 57-60

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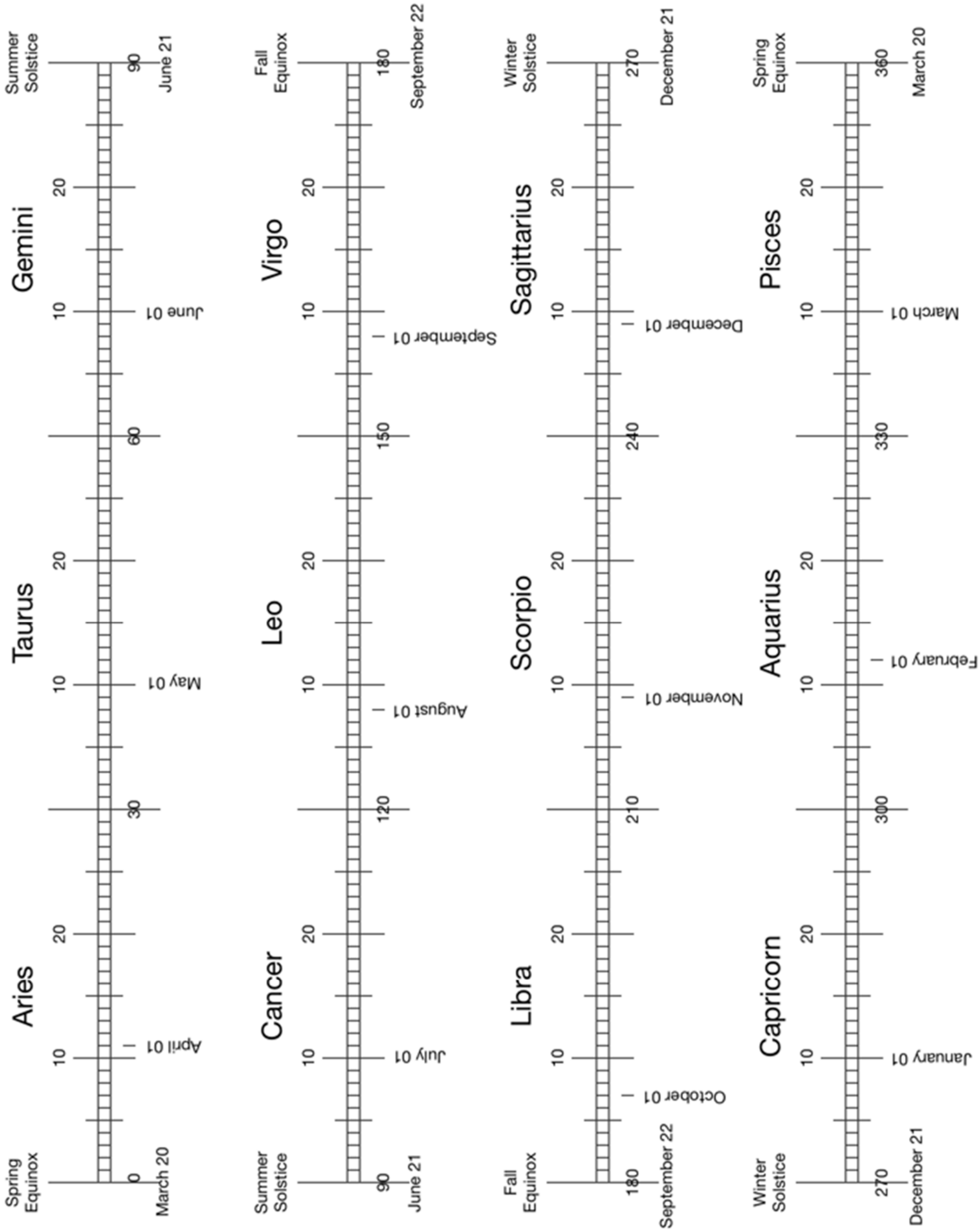
Morley, William H., Description of an Arabic Quadrant, Journal of the Royal Asiatic Society of Great Britain and Ireland, Vol 17 (1860), pp. 322-330

Appendix 1: Zodiac Chart



Appendix 2: Problems

1: Finding sine and cosine

- What is the sine of 45 degrees?
- What is the sine of 28 degrees?
- What is the cosine of 75 degrees?
- What is the cosine of 40 degrees?
- What is the angle that has a Cosine of 0.2?
- What is the angle that has a Sine of 0.4?

2: There is a tower in the town you are visiting, you wish to know the height. You pace out 30 feet from the base, and measure the angle to the top as 60 degrees. What is the height of the tower? (Hint: See functions list on page 5)

3: Find the angle of the Sun at noon for the first of August. Note: Pennsic is at 41 degrees latitude.

- Find the Sun's Zodiac position (see Appendix 1).
- Convert that to the correct angle on the quadrant
- Set the cord to that angle and read off the declination
- Subtract your latitude from 90, then add the declination to get the noon angle

4: You are traveling to Jerusalem and are camped near the city. You have the following information:

- It is the 14th of July.
- Jerusalem is located at: 31.8N 35.2E
- Mecca is located at: 21.4N 39.8E
- Find the following:
    - The angle of the Sun at the start of Asr
    - The angle of the Sun at the end of Asr
    - The approximate direction to Mecca

Appendix 3: Answers

1: Finding Sine and Cosine

- What is the Sine of 45 degrees? (42.5/60)
- What is the Sine of 28 degrees? (28/60)
- What is the Cosine of 75 degrees? (15.5/60)
- What is the Cosine of 40 degrees? (46/60)
- What is the angle that has a Cosine of 0.2? (12/60, 78.5)
- What is the angle that has a Sine of 0.4? (24/60, 23.5)

2: You have the distance to the tower (adjacent side), the angle to the top (angle), and wish to know the height (opposite side). Thus given opposite and adjacent you want to use the Tangent function of the angle:

Tangent a = opposite/adjacent

Or to get the answer:

Opposite = Tangent a * adjacent

Remember that all trig functions can be expressed as functions of sine and cosine. Tangent is the Sine of the angle divided by the Cosine of the angle:

Tangent a = Sine a / Cosine a
Or Tangent 60 = Sine 60 / Cosine 60
Or Tangent 60 = (52/60) / (30/60)
Tangent 60 = 0.87 / 0.5
Tangent 60 = 1.74

So the height of the tower is 1.74 * 30 = 52.2 (to be completely accurate, add the height of your eye, that’s where you are measuring from.)

3: August first the Sun is in Leo 8 or an angle of 128 degrees on the Zodiac.

- Take 128 degrees and subtract 90 to get 38 (Spring equinox to Summer solstice). Subtract this from 90 (remember the sun traveled 0 to 90 then back 38) to get 52.
- Reading from the declination/obliquity arc we get a declination of 18.5 degrees.
- As given, the latitude of Pennsic is approximately 41 degrees, so:

Height at Noon = declination + (90 - latitude)
Height at Noon = 18.5 + (90 - 41) = 67.5 degrees

Appendix 3: Answers

4: First we need the angle of the sun at noon

- 14 July has the sun in Cancer 24 at 114 degrees on the Zodiac.
- 114 degrees translates to an angle of 66 degrees on the quadrant (all the way to 90 then back 24 to 66)
- This gives a solar declination (using the obliquity arc) of 21.5 degrees
- which it turn gives us a angle of the sun at noon of

Height at Noon = declination + (90 - latitude)
Height at Noon = 21.5 + (90 - 31.8) = 79.7 degrees

Next, use the Asr lines on the quadrant to find the start and end elevations of the sun.
Start: 57.5 degrees
End: 40 degrees

Next, we need to estimate the direction to Mecca.

- First find the difference between the two latitudes, then the two longitudes
  Latitude difference is: 10.4 degrees
  Longitude difference is: 4.6 degrees

- Mark the horizontal quadrant line that is at 10.4 degrees down from 90 and the vertical line at 4.6 degrees.
- Place the cord on the point where they cross and read the angle: 21 degrees.

We know Mecca is South and East of us, so the direction to Mecca is 21 degrees East of South, or 159 degrees.